1. One Source vs. Two Source Solubility Problems
2. Challenging Solubility Problems
3. Prediction of Forming a Precipitate

One Source vs. Two Source Solubility Problems
One Source
Two Source

- Both ions come from the same salt (source)

$$
\mathrm{PbI}_{2(\mathrm{~s})} \leftrightharpoons \mathrm{Pb}^{2+}{ }_{(\mathrm{aq})}+2 \mathrm{I}^{-}(\mathrm{aq})
$$



$$
\begin{aligned}
K_{s p}= & {\left[\mathrm{Pb}^{2+}\right]\left[I^{-}\right]^{2} } \\
& (s)(2 s)^{2}=4 s^{3}
\end{aligned}
$$

- Ion concentrations are related through mole ratio

$$
\begin{aligned}
& \neq 1: 1 \text { ratio }=(s)(s)=s^{2} \\
& * 1: 2 \text { ratio }=(s)(2 s)^{2}=4 s^{3} \\
& * 1: 3 \text { ratio }=(s)(3 s)^{3}=27 s^{4} \\
& * 2: 3 \text { ratio }=(2 s)^{2}(3 s)^{3}=10 s^{s}
\end{aligned}
$$

- Both ions come from a different salt (source)

$$
\begin{aligned}
& \mathrm{Pb}\left(\mathrm{NO}_{3}\right)_{2(\mathrm{~s})} \leftrightharpoons \underset{x}{\mathbf{P b}^{2+}(\mathrm{aq})}+2 \mathrm{NO}_{3^{-}(\mathrm{aq})} \\
& \mathrm{KI}_{(\mathrm{s})} \leftrightharpoons \mathrm{K}^{+}{ }_{(\mathrm{aq})}+\mathrm{I}^{-}{ }_{(\mathrm{aq})} \\
& y \\
& \mathrm{PbI}_{2(\mathrm{ss})} \leftrightharpoons \underset{\mathrm{x}}{\mathrm{~Pb}^{2+}(\mathrm{aqq})}+\mathrm{y}^{2 \mathrm{I}_{\text {(aq) }}} \\
& K_{s p}=\left[P^{2+}\right][I]^{2}=(x)(y)^{2}
\end{aligned}
$$

- Related through $\mathrm{K}_{\text {sp }} \leftarrow$ from data booklet
- Ex: Find the $\left[\mathrm{I}^{-}\right]$if $\left[\mathrm{Pb}^{2+}\right]=4.5 \times 10^{-3} \mathrm{M}$. $x$

$$
\begin{aligned}
& K_{s p}=8.5 \times 10^{-9}=(x)(y)^{2} \\
& 8.5 \times 10^{-9}=\left(4.5 \times 10^{-3}\right)\left(y^{2}\right) \\
& y=\sqrt{\frac{8.5 \times 10^{-2}}{4.5 \times 10^{-3}}} \\
&=1.37 \times 10^{-3} \mathrm{M}=\left[I^{-}\right]
\end{aligned}
$$

2 source Challenging Solubility Problems

1. A solution has a concentration of calcium ions equal to $2.5 \times 10^{-2} \mathrm{M}$. What is the maximum concentration of $s^{2!}$ sulphate ions allowed to be added without causing precipitation?

$$
\begin{gathered}
\mathrm{K}_{5 p}=7.1 \times 10^{-5} \quad \mathrm{Sall}^{-5} \mathrm{CaSO}_{4(s)} \\
\left.\mathrm{CaSO}_{4(5)} \rightleftharpoons \mathrm{Ca}_{(\mathrm{aq})}^{2+}+\mathrm{SO}_{4}^{2-} \mathrm{Caq}\right) \\
\mathrm{K}_{5 p}=\left[\mathrm{Ca}^{2+}\right]\left[\mathrm{SO}_{4}^{2-}\right] \\
7.1 \times 10^{-5}=\left(2.5 \times 10^{-2}\right)\left[\mathrm{SO}_{4}^{2-}\right] \\
{\left[\mathrm{SO}_{4}^{20}\right]=2.8 \times 10^{-3} \mathrm{M}}
\end{gathered}
$$

2. Determine the maximum $\left[\mathrm{Na}_{2} \mathrm{CO}_{3}\right]$ that can exist in 1.0 L of $0.0010 \mathrm{M} \mathrm{Ba}\left(\mathrm{NO}_{3}\right)_{2}$ without forming a
A. $2.6 \times 10^{-12} \mathrm{M}$
B. $2.6 \times 10^{-9} \mathrm{M}$
C. $2.6 \times 10^{-6} \mathrm{M}$
D. $5.1 \times 10^{-5} \mathrm{M}$

$$
\begin{aligned}
& \mathrm{Na}_{2} \mathrm{CO}_{3}+\underset{\mathrm{Ba}\left(\mathrm{NO}_{3}\right)}{\mathrm{Ba}}{ }^{-} \\
& \times 10^{-9}=\left[\mathrm{Ba}^{2+}\right]\left[\mathrm{CO}_{3}^{2-}\right]^{-} \\
& \times 10^{-9}=(0,0010)\left[\mathrm{CO}_{3}^{2-}\right] \\
& {\left[\mathrm{CO}_{3}^{2-}\right]=2.6 \times 10^{-6} \mathrm{M}}
\end{aligned}
$$

$$
\begin{aligned}
K_{s p}=2.6 \times 10^{-9} & =\left[\mathrm{Ba}^{2+}\right]\left[\mathrm{CO}_{3}^{2-}\right] \\
2.6 \times 10^{-9} & =(0.0010)\left[\mathrm{CO}_{3}^{2-}\right]
\end{aligned}
$$


3. What is the maximum $\left[\mathrm{Sr}^{2+}\right]$ that can exist in a solution of $0.10 \mathrm{M} \mathrm{Na}_{2} \mathrm{SO}_{4}$ ?
A. $3.4 \times 10^{-7} \mathrm{M}$
B. $3.4 \times 10^{-6} \mathrm{M}$
C. $1.7 \times 10^{-6} \mathrm{M}$

$$
\mathrm{SrSO}_{y} \geqslant \mathrm{Sr}^{22}+\mathrm{SO}_{4}^{2-}
$$

D. $5.8 \times 10^{-4} \mathrm{M}$

$$
\begin{aligned}
K_{s p}=3.4 \times 10^{-7}= & {\left[\mathrm{Sc}^{2+}\right]\left[\mathrm{SO}_{4}{ }^{2-}\right] } \\
{\left[\mathrm{Sr}^{2+}\right] } & =\frac{3.4 \times 10^{-7}}{0.10 \mathrm{M}}=3.4 \times 10^{-6} \mathrm{M}
\end{aligned}
$$

$$
\begin{aligned}
& \mathrm{Sr}^{24}{ }_{(\mathrm{aq})}+\mathrm{Na}_{2} \mathrm{SO}_{4}(\mathrm{aa}) \rightleftharpoons 2 \mathrm{Na}^{+}{ }_{\text {(aq) }} \\
& \mathrm{Na}_{2} \mathrm{SO}_{4} \geqslant 2 \mathrm{Na}^{+}+\mathrm{SO}_{4}{ }^{2-} \\
& 0.10 \mathrm{M} \\
& 0.20 \mathrm{M} 0.10 \mathrm{M}
\end{aligned}
$$

4. When 100.0 m of $4.0 \times 10^{-2} \mathrm{M} \mathrm{CaCl}_{2}$ is added to 150.0 mL of $2.9 \times 10^{-2} \mathrm{M} \mathrm{NaOH}$, a precipitate just starts to form. What is the $\mathrm{K}_{\text {sp }}$ of this precipitate?

- Write a (balanced) double replacement reaction.
- What is the possible precipitate? Write the $\mathrm{K}_{\text {sp }}$ expression.

$$
\mathrm{Ca}(\mathrm{OH})_{2} \rightleftharpoons \mathrm{Ca}^{2+}+2 \mathrm{OH}^{-}
$$

$$
K_{\text {sp }}^{\text {sion }}:\left[\mathrm{Ca}^{2+}\right][\mathrm{OH}-]^{2}
$$

$C_{1} V_{1}=C_{2} V^{2} \quad$ Calculate the diluted concentrations of each ion.

$$
\begin{aligned}
& \text { [ } \mathrm{CaCl}_{2} \text { ] } \\
& c_{1} v_{1}=c_{2} v_{2} \\
& \left(4.0 \times 10^{-2}\right)(100.0)=\left(c_{2}\right)(250.0) \\
& \mathrm{C}_{2}=16 \times 10^{-2} \mathrm{M}:\left[\mathrm{CaCl}_{2}\right] \\
& \mathrm{CaCl}_{1.6 \times 10^{-2}}=\binom{\mathrm{Ca}^{2+}+1}{1.6 \times 10^{-2} \mathrm{M}}
\end{aligned}
$$

[ NaOH ]

$$
c_{1} v_{1}=c_{2} v_{2}
$$

$$
\begin{aligned}
& (2.9 \times 10-2)(150)=\left(C_{2}\right)(250) \\
& \mathrm{C}_{2}=1.7 \times 10^{-2} \mathrm{M}=[\mathrm{NaOH}] \\
& \mathrm{NaOH} \geqslant \mathrm{Na}^{+} \mathrm{OH}^{-} \\
& 1.7 \times 10^{-2}-2
\end{aligned}
$$

$$
\begin{aligned}
k_{\text {sp }} & =\left(1.6 \times 10^{-2}\right)\left(1.7 \times 10^{-2}\right)^{2} \\
& =4.6 \times 10^{-6}
\end{aligned}
$$ (liquid)

5. Up to 15.0 g of $\mathrm{BaCl}_{2}$ can be dissolved in 2.5 L of $\mathrm{Al}_{2}\left(\mathrm{SO}_{4}\right)_{3}$ without a precipitate being formed. Find $\left[\mathrm{Al}_{2}\left(\mathrm{SO}_{4}\right)_{3}\right]$.

- Write the double replacement reaction. (What is the solute? What is the solvent?)

$$
\left.\mathrm{BaCl}_{2(s)}^{=}+\mathrm{Al}_{2}\left(\mathrm{SO}_{4}\right)_{3(a)}^{=} \stackrel{\mathrm{Ba}}{\stackrel{\mathrm{SO}}{4}(\mathrm{ss})}\right\}+2 \mathrm{AlCl}_{3(9)}
$$

- What is the possible precipitate that could be formed? Write the $\mathrm{K}_{\mathrm{sp}}$ expression and determine its value from the data booklet.

$$
K_{s p}=\left[\mathrm{Ba}^{2+}\right]\left[\mathrm{SO}_{4}^{2-}\right]=1.1 \times 10^{-10}
$$

- Looking at the $\mathrm{K}_{\text {sp }}$ expression, is there an ion concentration value that could be determined $2\left(\left[\mathrm{Al}_{2}\left(\mathrm{SO}_{4}\right)_{3}\right]\right)$

$$
\begin{aligned}
& (0.029)\left[\mathrm{SO}_{4}^{2-}\right]=1.1 \times 10^{-10} \\
& {\left[\mathrm{SO}_{4}{ }^{2-}\right]=3.8 \times 10^{-9} \mathrm{M}} \\
& \text { Worksheet 3.3 \#1-7 } \\
& \begin{array}{rl}
\mathrm{Al}_{2}\left(50_{4}\right)_{3} & 2 \mathrm{Al}^{3+}+ \\
1.3 \times 15^{-9} \mathrm{M}
\end{array} \quad \begin{aligned}
& 2- \\
& 3.8 \times 10^{-9}
\end{aligned}
\end{aligned}
$$

When two different solutions are mixed, we can predict whether a precipitate will form. Th $\mathrm{K}_{\text {sp }}$ value represents the maximum product of the ion concentrations in a saturated solution.

If an equilibrium is not present in solution, then we calculate a trial ion product (TIP) - (also called a trial Kip value or reaction quotient, Q)

If Trial $K_{\text {sp }}>$ Actual $K_{\text {sp }}$ - a precipitate forms. (OVersaturated)
If Trial $K_{\text {sp }}<$ Actual $K_{\text {sp }}$ - no precipitate forms. (Under saturated)
If Trial $K_{s p}=$ Actual $K_{\text {sp }}$ - the solution is saturated.

$$
\begin{gathered}
\mathrm{X}_{2} \mathrm{Y}_{(\mathrm{s})} \leftrightharpoons 2 \mathrm{X}^{+}+\mathrm{Y}^{2-} \\
\mathrm{K}_{\text {sp }}=\left[x^{+}\right]^{2}\left[y^{2-}\right]
\end{gathered}
$$

Example.
(1) Will a precipitate form when 23 mL of $0.020 \mathrm{M} \mathrm{Na}_{2} \mathrm{CO}_{3}$ is added to 12 mL of $0.010 \mathrm{M} \mathrm{MgCl}_{2}$ ?

- Write a balanced equation. What is the precipitate that will potentially form? (Use the solubility table)

$$
\xrightarrow[\text { What are the concentrations of each of these ions? }]{\mathrm{Na}_{2} \mathrm{CO}_{3} \text { (as) }+\mathrm{MgCl}_{\text {(au) }}+2 \mathrm{NaCO}_{3} \text { (s) }}
$$

$$
\begin{gathered}
{\left[\mathrm{MgCl}_{2}\right]} \\
(0.010)(12)=\mathrm{C}_{2}(35) \\
\mathrm{C}_{2}=0.0034 \mathrm{M}=\left[\mathrm{MgCl}_{2}\right] \\
\mathrm{MgCl}_{2}=\mathrm{Mg}^{2+}+2 \mathrm{Cl}- \\
0.0034 \mathrm{M}
\end{gathered}
$$

$$
(0.020)(23)=c_{2}(35)
$$



- Calculate the value of TIP (Trial $\mathrm{K}_{\text {sp }}$ )

$$
\begin{aligned}
\text { Trial } K_{s p} & =\left[\mathrm{Mg}^{2+}\right]\left[\mathrm{CO}_{3}^{2-}\right] \\
& =(0.0034 \mathrm{M})(0.013 \mathrm{M}) \\
& =4.4 \times 10^{-5}
\end{aligned}
$$

- Compare the TIP (Trial $\mathrm{K}_{\mathrm{sp}}$ ) with the real $\mathrm{K}_{\mathrm{sp}}$. Will a precipitate form?

$$
\text { Actual } K_{\text {sp }}=6.8 \times 10^{-6}
$$

$$
\begin{aligned}
& \text { Trial }>\text { Actual } \\
& \therefore \text { Yes, apt will } \text { form? }
\end{aligned}
$$

(2) Will a precipitate form when 8.5 mL of $6.3 \times 10^{-2} \mathrm{M}$ lead (II) nitrate is added to 1.0 L of $1.2 \times 10^{-3} \mathrm{M}$ sodium iodate?

$$
\begin{aligned}
& \underline{\mathrm{Pb}}\left(\mathrm{NO}_{3}\right)_{2(\mathrm{aq})}+\mathrm{NaIO}_{3}(\mathrm{aq}) \geqslant \mathrm{Pb}_{\left(1 \mathrm{O}_{3}\right)_{2(5)}}+\mathrm{NaNO} \\
& {\left[\mathrm{~Pb}\left(\mathrm{NO}_{3}\right)_{2}\right]} \\
& \left(6.3 \times 10^{-2}\right)(8.5)=C_{2}(1008.5) \\
& \left(1.2 \times 10^{-3}\right)(1000)=C_{2}(1008.5) \\
& C_{2}=5.31 \times 10^{-4} \mathrm{M} \\
& C_{2}=1.18 \times 10^{-3} \mathrm{M} \\
& \mathrm{NaIO}_{3} \geqslant \mathrm{~N} \mathrm{~K}^{+}+\begin{array}{l}
10_{3}^{-} \\
1.8 \times 10^{-3} \mathrm{M}
\end{array} \\
& \text { Trial } K_{s p}=\left[\mathrm{Pb}^{2+}\right]\left[10_{3}\right]^{2} \\
& =\left(5.31 \times 10^{-4}\right)\left(1.8 \times 10^{-3}\right) \\
& =7.6 \times 10^{-10} \quad \text { Trial }>\text { Actual } \\
& \text { Actual } K_{s p}=3.7 \times 10^{-13} \\
& \therefore \text { opt will form }
\end{aligned}
$$

(3) Will a precipitate form when 1.5 mL of $4.5 \times 10^{-3} \mathrm{M}$ ammonium bromate is added to 120.5 mL of $2.5 \times 10^{-3} \mathrm{M}$ silver nitrate?

$$
\begin{aligned}
& \mathrm{NH}_{4} \underline{\mathrm{BrO}}_{3(\mathrm{aq})}+\underline{\mathrm{AgNO}}_{3(\mathrm{aq})} \rightleftharpoons \underbrace{\mathrm{Ag}_{3-\mathrm{O}_{3}}^{(5)}+\mathrm{NH}_{4} \mathrm{NO}_{3}(\mathrm{aq})} \\
& {\left[\mathrm{AgNO}_{3}\right]} \\
& \left(2.5 \times 10^{-3}\right)(120.5)=c_{2}(122.0) \\
& \left(4.5 \times 10^{-3}\right)(1.5)=C_{2}(122.0) \\
& c_{2}=2.47 \times 10^{-3} \mathrm{M} \\
& C_{2}=5.53 \times 10^{-5} \\
& \underbrace{\mathrm{AgNO}_{3}}_{\text {Trial } \mathrm{Ksp}}=\left[\mathrm{Ag}^{+}\right]\left[\mathrm{BrO}_{3}^{-}\right] \\
& \mathrm{NH}_{4} \mathrm{BrO}_{3} \rightleftharpoons \mathrm{NH} / 4+\begin{array}{l}
+\mathrm{BrO}_{3}^{-} \\
5.53 \times 10^{-5} \mathrm{M}
\end{array} \\
& =\left(2.47 \times 10^{-3}\right)\left(5.53 \times 10^{-5}\right) \\
& =1.37 \times 10^{-7} \\
& \text { Actual Ks }=5.3 \times 10^{-5} \\
& \text { Trial < Actual } \\
& \therefore \text { ppm will not form }
\end{aligned}
$$

